and it is clear that the curly bracket in (A8) is one order of magnitude smaller than the leading terms, provided that $\beta\Gamma \gg 1$. Using (A4), the calculation of X_f in (63) leads to

$$X_f=\left(\frac{c^2}{2\pi}\right)^2\left[-\left(\frac{1}{\pi}\arctan\frac{|c\xi_0|}{\beta\Gamma}\right)^2\frac{1}{\pi}\frac{\beta\Gamma}{\beta^2\Gamma^2+c^2\xi_0^2}+\frac{2}{\pi}\left(\arctan\frac{c|\xi_0|}{\beta\Gamma}\right)\frac{\beta\Gamma c|\xi_0|}{(\beta^2\Gamma^2+c^2\xi_0^2)^2}\right]. \qquad \text{(A9)}$$

Here we have already dropped a third term, which is half the second term in the curly bracket in (A8). For $c\xi_0 \ll \beta\Gamma$, X_f is of the order of $c^4(c\xi_0)^2/(\beta\Gamma)^3$ and much smaller than X_f is of the order $c^2\beta\Gamma/\xi_0^2$ and much larger than the other diagrams.

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Spin-Wave Relaxation in EuS†

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As part of a study of spin-wave relaxation, spin-wave linewidths have been measured in EuS. EuS is a good candidate for this study because of its high saturation magnetization, $4\pi M = 14\,000$ G at $T = 0$ °K and small exchange energy which is reflected in its Curie temperature, $T_C = 16$ °K. The linewidths due to three-spin-wave dipole-dipole interactions are large ranging up to 100 Oe. The theory of three-spin-wave relaxation processes explains the linewidths observed both in relation to different spin waves at a fixed temperature and temperature dependence for a given spin wave over the range measured, i.e., 2–6 °K.

I. INTRODUCTION

Materials which have a high magnetization offer a new area for the study of spin-wave relaxation effects. Since the magnetic dipole-dipole interaction in these materials is strong, we expect that relaxation processes associated with this interaction will contribute significantly. EuS which has a saturation magnetization of 14 000 G at $T = 0$ °K is a good candidate for this study. In this compound, europium is divalent with ground-state parameters $S=\frac{7}{2}$, $L=0$, and $g\sim 2.00$. The S-state character of this ion insures that there is little coupling to the lattice and that the magnetic anisotropy is low. EuS is ferromagnetic with a Curie temperature of 16 °K.[1] This low value of the critical temperature reflects a relatively small exchange parameter which, together with a large dipolar interaction, determines many of the properties of this material. Our sample was obtained from Dr. D. Teaney and Dr. M. W. Schafer of the IBM Watson Research Laboratory. It was formed into a sphere of diameter 0.79 mm. The surface of the sample was polished with a 10-μ grit, which was sufficient for our experiments. Finally, the sample was annealed in a vacuum at 1000 °K for 24 h, and slowly cooled to remove strains from the crystal.

II. EXPERIMENTAL ARRANGEMENT

The spin-wave relaxation time in this material was measured by the parallel-pump technique. For this measurement, the sample is placed in the high-magnetic-field position of a microwave cavity. This in turn is situated in a dc magnetic field which serves to align the magnetization of the sample parallel to the microwave field. The microwave power level to the cavity is raised until the spin waves begin to absorb power. This threshold power at which the spin waves begin to absorb power and grow in amplitude can then be related to the spin-wave relaxation time or effective linewidth.[2] For this experiment, we used a microwave frequency of 16.7 GHz. Since the spin waves are excited in pairs as a standing wave, their frequency is one-half this value. Varying the applied dc field changes the energy of the spin waves, and brings various parts of the long-wavelength or low-wave-vector-$\vec{k}$ end of the spin-wave spectrum into the range of the apparatus. In general, the spin wave of lowest $|\vec{k}|$ at a given frequency is the first to become unstable and consequently is the one whose linewidth is measured. The spin waves which can be studied in this manner are shown in Fig. 1 which shows the low-$\vec{k}$ portion of the spin-wave dispersion relation. In the present experiment, we measured spin-wave linewidths over the entire range shown by the dashed line.

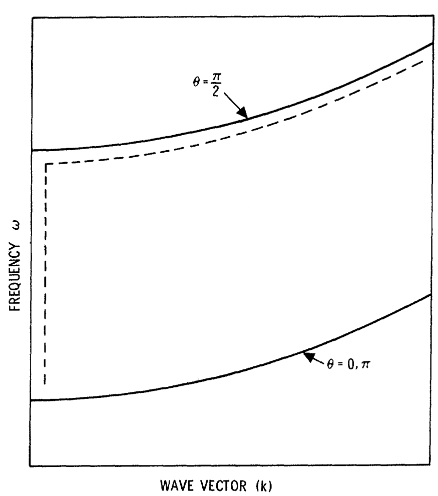

FIG. 1. Low-$\vec{k}$ portion of the spin-wave spectrum, showing (dashed line) the region which can be investigated by the parallel-pump experiment.

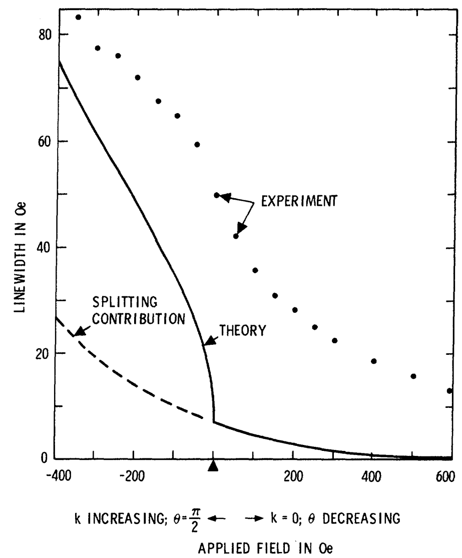

FIG. 2. Linewidth as a function of applied magnetic field measured from 5200 Oe. The theoretical curve is the sum of the three-magnon confluence and splitting contributions. The onset of the splitting contribution as k increases from zero is clearly seen.

III. DATA AND INTERPRETATION

The spin-wave spectrum of Fig. 1 is characterized by the parameter k, the magnitude of the wave vector, and θ, the angle between $\vec{k}$ and the magnetization of the sample. In Fig. 1 we can see that there are two regions of the spectrum that we can investigate. One is the top of the curve, i.e., the low-k, $\theta=\frac{1}{2}\pi$ portion, and the other is the left-hand side, i.e., the $k=0$, $0<\theta<\frac{1}{2}\pi$ portion. The point where these regions join, i.e., $k=0$, $\theta=\frac{1}{2}\pi$ is referred to as the corner position. Data have been obtained in both regions. Since the measurements are made by determining linewidths as the applied dc magnetic field is varied, the data are presented in this manner. For clarity, the regions of the spin-wave dispersion curve corresponding to particular values of the applied field are indicated.

The data of linewidth versus applied field at 4.2 °K are presented in Fig. 2 for all spin waves whose relaxation was measured. At this temperature the magnon populations due to thermal excitation are low, especially for the large-k high-energy magnons which have a large density of states and contribute strongly to the relaxation. We expect, therefore, that lower-order processes, which involve few magnons, will dominate. The order of a relaxation process refers to the number of magnons involved, i.e., decay of one mode into another is a two-magnon process. The two-magnon process is the lowest order to be considered, but it is ineffective in relaxing the parallel-pump mode.[3] The three-magnon processes are the next to be considered. These are two types, confluence and splitting. The confluence process involves the combining of an excited and a thermal magnon to form a resultant. This process has been treated in detail by Sparks *et al.*[4] and Schlömann.[5] At constant temperature it is found that the linewidth due to this process is directly proportional to the magnitude of the wave vector k. From the dispersion relation, ω is related to $(H_i - Dk^2)$; H_i is the value for the internal field of the sample. We then find that the linewidth varies as $(H_0 - H)^{1/2}$, where H_0 is the value of the applied field for excitation of the corner magnon with $k=0$, $\theta = \frac{1}{2}\pi$, and H is the value of the field applied for the measurement. This behavior can be seen in the theoretical curve on Fig. 2.

The splitting process involves the decay of an excited magnon into two others. This problem has also been treated by Sparks *et al.* and Schlömann, but only in certain restricted limits. A calculation of the relaxation rate due to this process over the range of the present experiment has been performed by the author.[6] From this treatment, the contribution of the confluence process to the linewidth has been calculated and is also shown in Fig. 2. All of these calculations involve only simple spin-wave theory with the inclusion of the dipole-dipole interaction terms up to third order. Relaxation due to four-magnon processes which arise from the dipole-dipole and exchange interactions has also been considered. From the calculations of Pincus, Sparks, and LeCraw,[7] these were found to be completely negligible over the range of this experiment. These terms are all isotropic in a cubic structure like that of EuS which also has low magnetocrystalline anisotropy because of its S-state character. Experimentally, the linewidths were found to be isotropic, so no mention is made of any particular orientation. Comparing the experimental data with the results of the calculations mentioned above, we see that in magnitude and shape the fit appears to be reasonable in the light of the following considerations. Problems involving the sensitivity of our apparatus, especially at the points where the linewidth is lowest, can cause a discrepancy of the type seen here. If the absorption signal at the onset of the instability is low and saturates to a low value of total absorbed power, more power must be added to excite more spin waves until the absorption is seen. In this way we excite not only the magnons represented in Fig. 1, but also a group of finite width which extends into the higher-k portion of the spectrum at the same frequency. This process leads to a higher measured value for the linewidth. This is somewhat like the effects to be seen in the case where the magnons are coupled, as has been treated by Sage.[8] There is, however, no evidence for any internal coupling of magnons in this material. All of these excited magnons, however, should relax according to the same three-magnon processes. If this supposition is correct, we have another way to test the theory. We can examine the temperature dependence of the spin-wave linewidths. Owing to the population factors involved, three-boson processes in the high-temperature limit, $\hbar\omega < kT$, have a linewidth that is proportional to the absolute temperature. Choosing three representative spin waves, we plot their linewidths as a function of temperature in Fig. 3. The spin waves chosen represent three regions of the spectrum: (1) $k<0$, $\theta = \frac{1}{2}\pi$ occurring 200 Oe below the corner, (2) $k=0$, $\theta=\frac{1}{2}\pi$ or the corner of Fig. 1, and (3) $k=0$, $\theta < \frac{1}{2}\pi$ occurring 600 Oe above the corner. Figure 3 shows the linear relation between linewidth and temperature for these representative spin waves. This temperature dependence, as mentioned previously, is characteristic of three-magnon process. In case 1, which corresponds to $k = 3.0\times10^6\ \text{cm}^{-1}$, the slope of the curve is the greatest of the three in accordance with theory which

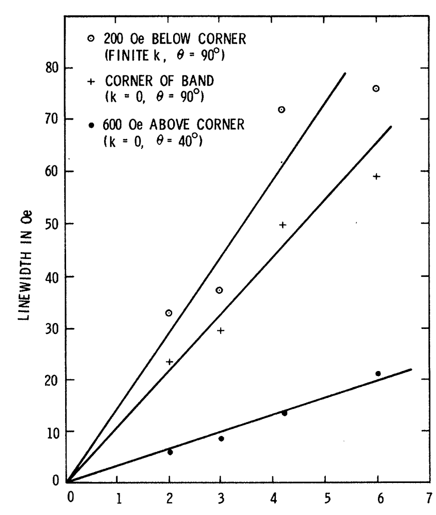

FIG. 3. Linewidth for three representative spin waves at low temperatures, pointing out the linear relation between the measured linewidth and temperature in this region.

shows that the slope of linewidth versus temperature for the dominant three-magnon confluence process is proportional to k. In magnitude, the slope is 30% higher than predicted by theory. Cases 2 and 3 are much higher than theory, reflecting the discrepancies in their region of Fig. 2. The slopes observed for these cases point to the excitation of higher-k spin waves as mentioned previously with case 2 having a higher slope corresponding to the greater range of k available at the energy of the corner position. Figure 3 also shows the three curves intercepting the temperature axis at $T=0\,^\circ$K. In the treatment of Sparks,[9] which does not employ the high-temperature approximation, it is shown that a $T=0\,^\circ$K intercept on a plot of this type is only a good approximation for $\vec{k}$ greater than a value given in his theory. For the case of EuS this value corresponds to an intercept of $0.5\,^\circ$K for case 1, owing to the breakdown of the high-temperature approximation for the spin waves which interact with the excited state. For cases 2 and 3, it appears, from the magnitude of the linewidth, that the spin waves actually excited in this experiment have a value of k comparable to case 1 or have a value of θ different from 90°. In this case, the high-temperature approximation for all spin waves involved in the relaxation can be valid and the $t=0\,^\circ$K intercept seen in Fig. 3 can be understood.

IV. CONCLUSION

Although experimental difficulties exist, it appears that the spin-wave relaxation in EuS is explained by three-magnon processes. These processes can account for the variation in linewidth with spin-wave parameters as the spin-wave spectrum is swept. They also predict the linear temperature dependence for the linewidth of a given spin wave, which is observed from 2 to $6\,^\circ$K. This leads us to the conclusion that simple spin-wave theory, with the inclusion of two- and three-mangon terms from the dipole-dipole interaction, provides a satisfactory explanation of spin-wave relaxation in the low-$\vec{k}$ portion of the spin-wave spectrum even in the high-magnetization low-exchange materials where the interactions are very strong.

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Approach to Magnetic Saturation in the Vicinity of Impurity Atoms

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Ritz-method calculations are used to estimate the angles between applied magnetic fields of the order of 10^3 Oe and the magnetization in the vicinity of W or Nd impurity atoms in Fe or Ni. It is assumed that the impurity atom is substitutional and that its main effect is the magnetostriction, due to the mechanical strain caused by introducing an atom having a different size from that of the other atoms in the lattice, but it is shown that magnetostatic effects, caused by introducing a nonmagnetic atom, might also be important. Radii of free atoms are used, and because of this and other approximations used in the calculations of the various energy terms, the theoretical angles are only a crude approximation; but they do come to within a factor of 2 or so of the experimentally measured angles.

INTRODUCTION

In a previous publication[1] it has been argued that the direction of the magnetization in the immediate vicinity of impurity atoms in soft ferromagnetic materials, such as Fe or Ni, differs considerably from its direction in most of the ferromagnetic material when a magnetic field of the order of 10^3 Oe is applied. This was essentially an *ad hoc* interpretation of the experimental results of Ben-Zvi *et al.*[2] on recoiling nuclei embedded in Fe or Ni foils. According to this experiment,[2] the field